LAW OF SINES

Ch.6  Lesson 4
Target  TSWBAT: Use the law of sines to find missing angles and sides of a non-right triangle. Be able to distinguish the ambiguous case.

Agenda  Warm-Up/Homework Check
          turn in pg. 557 inverse functions
          Lesson: Law of sines, cases, examples

Purpose  BAT: look ahead and predict what will happen, solve the angles and sides of any triangle.

Evaluation  3-2-1
Warm-Up:

1. Find the height of the triangle given below using trig ratios

\[
\sin 46^\circ = \frac{h}{14} \\
h = 14 \sin 46 \\
h = 10.1
\]

2. Find the area of the triangle

\[
\sin 90^\circ = \frac{\sin 44}{14} \\
x = 14 \sin(44) \\
x = 9.7
\]

\[
\sin 29^\circ = \frac{\sin 61}{10.1} \\
y = 10.1 \sin 61 \\
y = 18.2
\]

\[
b = 9.7 + 18.2 \\
b = 27.9
\]

\[
A = \frac{1}{2} b \cdot h = \frac{1}{2}(27.9)(10.1) \\
A = 140.9 \text{ units}^2
\]
Law of Sines

Used to solve triangles that are not right triangles

Used when given ASA or SAA, and SSA

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]
Example:

Solve \( \triangle ABC \) if \( A = 33^\circ \), \( B = 105^\circ \), and \( b = 38 \)
**Obtuse Triangles**

1. In a triangle, the sum of the interior angles is 180°.
2. No triangles can have two obtuse angles.

Used when the information provided gives _SSA_ (A, a, & b)

Three possible cases:
1) There is no triangle (a is too short)
2) There is exactly one triangle (right triangle, or like previous)
3) There are two possible triangles
The cases for Acute Angles

Types of triangles made from 3rd side

When the given angle is ACUTE (A < 90°)

1. No Solution

2. One Solution

3. Two Solutions
The cases for OBTUSE angles

When the given angle is OBTUSE (A > 90°)

1. No Solution
2. One Solution
1) Use the Law of Sines to find 2nd angle, test to see if it works, is the sum less than 180?  

\[ \text{yes}\]  

2) find the 3rd angle \((C)\)  

3) Check if there is a second angle that is valid by subtracting the angle you got in step 1 from 180, \((180 - B)\), and then adding it to the original angle, is the sum less than 180?  

\[ \text{no}\]  

\[ \text{solution #1 (__,__,__)} \]  

\[ \text{no solution} \]  

\[ \text{yes}\]  

4) find the new 3rd angle \((C)\)  

\[ \text{solution #2 (__,__,__)} \]
What will it have?

Find all solutions for the triangle.

\[ A = 54^\circ, a = 18, b = 20 \]

1) Use the Law of Sines to find 2nd angle, test to see if it works, is the sum less than 180?

yes

no

3) Check if there is a second angle that is valid by subtracting the angle you got in step 1 from 180, \((180 - B)\), and then adding it to the original angle, is the sum less than 180?

no solution

2) find the 3rd angle \((C)\)

yes

no

solution #1 (__,__,__)

solution #2 (__,__,__)

4) find the new 3rd angle \((C)\)
A = 41°, a = 40, b = 89

\[
\frac{\sin 41°}{40} = \frac{\sin B}{89} \\
\sin B = \frac{89 \sin 41°}{40} \\
\sin B = 1.5
\]

No Solution

\[1.5 > 1\]

\[-1 < \sin B < 1\]

A = 45°, a = 9√2, b = 9

Law of Sines \rightarrow B = 30°

180 - 30 = 150

150 + 45 = 195

195 < 180

1 Solution

B = 30°, C = 105°

\[c = 17\]

(30°, 105°, 17°)
Evaluation:

1. Find all solutions for the triangle.
   \[ A = 29°, a = 11, b = 20 \]

Practice!
pg. 316 # 19, 21, 24
pg. 331 #19, 20, 23
pg. 324
#18, 19, 20, 22, 24, 26

2 problems from the rest of level
4 pages 317, 325, 331
Day 2
Target  
TSWBAT: Use rules to predict how many cases. Find the ambiguous case of SSA triangles, Solve triangle word problems

Agenda  
Warm-Up/Homework Check  
Lesson: Formulas for Prediction  
Outline to solving  
Law of sines, examples  
Word Problems for Sine and Cosine

Purpose  
BAT: look ahead and predict what will happen, solve the angles and sides of any triangle

Evaluation  
3–2–1
What will it have?

Find all solutions for the triangle.

\[ A = 29^\circ, a = 11, b = 20 \]

1) find \( B_2 = 180^\circ - B \) \( (B_2, C_2, A) \)
2) find \( C_2 \) \( 180 - B_2 - A = C_2 \)
3) find \( C_2 \) Use Law of Sines
   1) \( B_2 = 180^\circ - 61.8^\circ = 118.2^\circ \)
   2) \( C_2 = 180 - 118.2^\circ - 29^\circ = 32.8^\circ \)
3) \( \frac{\sin 29^\circ}{11} = \frac{\sin 32.8^\circ}{C_2} \)
   \[ C_2 = \frac{11 \sin 32.8^\circ}{\sin 29^\circ} \]
   \[ C_2 = 12.3 \]

(12.3, 118.2, 32.8°)
\[ a^2 = b^2 + c^2 - 2bc \cos A \]

- 0 solutions: \( a < b \sin A \)
- 1 solution: \( a = b \sin A \)
- 2 solutions: \( b \sin A < a < b \)

\[ A = 123^\circ \]
\[ b = 16 \]
\[ a = 22 \]

\[ \sin 123^\circ = \frac{\sin B}{22} \]
\[ \sin B = \frac{16 \sin 123^\circ}{22} \]
\[ B = \sin^{-1} \left( \frac{16 \sin 123^\circ}{22} \right) \]
\[ B = 37.6^\circ \]

\[ C = 180^\circ - 37.6^\circ - 123^\circ \]
\[ C = 19.4^\circ \]

\[ \frac{\sin 19.4^\circ}{c} = \frac{\sin 123^\circ}{22} \]
\[ c = \frac{22 \sin 19.4^\circ}{\sin 123^\circ} \]
\[ c = 8.7 \]

\[ B_2 = 180^\circ - 37.6^\circ = 142.4^\circ \]

\[ 142.4^\circ + 123^\circ < 180^\circ \]
\[ 265.4^\circ < 180^\circ \]

No solution
Example 46: Two tugboats that are 120 ft apart pull a barge, as shown. If the length of one cable is 212 ft and the length of the other is 230 ft, find the angle formed by the two cables.

\[ 230^2 = 120^2 + 212^2 - 2 \cdot 120 \cdot 212 \cdot \cos \theta \]
\[ 52900 = 14400 + 44944 - 50880 \cos \theta \]
\[ 52900 = 59344 - 50880 \cos \theta \]
\[ -6444 = -50880 \cos \theta \]
\[ 0.1266 = \cos \theta \]
\[ \theta = \cos^{-1}(0.1266) 
\]
\[ x = 82.7^\circ \]

\[ \frac{\sin \theta}{120} = \frac{\sin 82.7^\circ}{230} \]
\[ \sin \theta = \frac{120 \sin 82.7^\circ}{230} \]
\[ \theta = \sin^{-1}\left(\frac{120 \sin 82.7^\circ}{230}\right) \]
\[ \theta = 31.2^\circ \]

The angle between cables is 31.2° apart.
Summarizing
Evaluation:

1. Find all solutions for the triangle.
   \[ A = 29^\circ, \ a = 11, \ b = 20 \]

Practice! Quiz Tomorrow!

Study the levels you need, especially level 1, learn formulas!